Invesco Perspective

Authored by Invesco Quantitative Strategies

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For General Institutional Investor Use

A New Way to Measure Risk: Risk-K

The Definition of Risk

In the world of finance, standard deviation has become the accepted measure of risk. Outside the world of finance, if you ask people how they define risk they will usually reply with the following characteristics relating to risk:

1) The likelihood of having a downside event (who worries about upside risk?)
2) The magnitude and duration of the possible downside event

For example, if there is an 80% chance that I will lose my entire investment if I invest in a movie, then it is doubtful that I will want to make that investment, even if the upside potential is great.

The Finance World's Most Common Measure of Risk

However, in the world of finance, these logical definitions of risk do not apply. Instead we use standard deviation of arithmetic excess returns as the common measure of risk. This is called "tracking error."

Let's use an example to see how tracking error is measured and what it is telling us. The example below is a series of 12 months of returns from an unnamed manager.

Table A: Standard Deviation Calculation Example

Month Number	Gross Return	Benchmark Return	Current Month Excess Return	(Current Month Excess Return) - (Average of all Months' Excess Returns)	((Current Month Excess Return) - Average of All Months' Excess Returns))^2
1	3.2	0.5	2.7	(1.4)	1.96
2	2.4	0.5	1.9	(2.2)	4.84
3	6.2	0.5	5.7	1.6	2.56
4	1.2	0.5	0.7	(3.4)	11.56
5	5.3	0.5	4.8	0.7	0.49
6	3.5	0.5	3.0	(1.1)	1.21
7	8.3	0.5	7.8	3.7	13.69
8	4.1	0.5	3.6	(0.5)	0.25
9	3.5	0.4	3.1	(1.0)	1.00
10	4.2	0.5	3.7	(0.4)	0.16
11	8.8	0.5	8.3	4.2	17.64
12	4.2	0.4	3.8	(0.3)	0.09
Average	4.6	0.5	4.1		4.62
Standard Deviation			2.15		
Square Root					2.15
Total Return	70.6	6.0	64.7		
Annualized Standard Deviation note: *sqrt(12)		7.5		7.5	
Information Ratio			8.7		

The tracking error of this manager's excess returns is 7.5% annualized. Incredibly, a manager that was apparently taking less than 8% risk per year outperformed its benchmark by more than 64%.

The traditionally applied measure of tracking error appears to understate risk in this case. Below, we consider three problems with the risk and excess return calculation creating this situation and propose adjustments to create a new risk measure, "Risk-K."

PROBLEM #1—The Assumed Mean in the Standard Deviation Calculation

To define the portfolio of all active managers taken as a whole, Bill Sharpe has often said, "If you think of active managers taken as a collective group, their portfolio has to look like the broad based market" (maybe the Russell 3000 Index). So, if active managers collectively look like the index, they have to perform in line with the index over time, before deducting fees and transaction costs. Therefore, it is reasonable to assume the average active manager will add no value. The current standard deviation calculation assumes that the mean excess return is the mean excess return achieved during the given time period. Using Bill Sharpe's logic, a better approach would be to assume a mean excess return over time of 0% for active managers.

RECOMMENDATION #1: Assume that the mean excess return for the average active manager over a given time period is zero.

Using this assumption, we could re-calculate annualized deviation from zero as shown below:

Table B: Deviation Calculation Assuming Mean Return = 0

Month Number	Gross Return	Benchmark Return	Current Month Excess Return	(Current Month Excess Return) - 0	((Current Month Excess Return) - 0)^2
1	3.2	0.5	2.7	2.7	7.29
2	2.4	0.5	1.9	1.9	3.61
3	6.2	0.5	5.7	5.7	32.49
4	1.2	0.5	0.7	0.7	0.49
5	5.3	0.5	4.8	4.8	23.04
6	3.5	0.5	3.0	3.0	9.00
7	8.3	0.5	7.8	7.8	60.84
8	4.1	0.5	3.6	3.6	12.96
9	3.5	0.4	3.1	3.1	9.61
10	4.2	0.5	3.7	3.7	13.69
11	8.8	0.5	8.3	8.3	68.89
12	4.2	0.4	3.8	3.8	14.44
Average	4.6	0.5	4.1		21.36
Standard Deviation			2.15		
Square Root					4.62
Total Return	70.6	6.0	64.7		
Annualized Deviation from Zero					16.01

Calculated this way, as an annualized deviation from zero, the tracking error would be a more meaningful 16.01%.

In the extreme, as shown below, if the unnamed active manager had outperformed by exactly 4.1% every month, using the currently accepted calculation of risk, its standard deviation would be zero. This would imply the manager took no risk! However, common sense would lead us to the conclusion that the manager would have had to have taken risk to achieve those excess returns. Its monthly deviation from zero would have been 4.1%, and by multiplying by the square root of 12, would be 14.2% annualized.

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Table C: Deviation Calculation Assuming Mean Return = 0 vs. Standard Deviation Calculation

Month Number	Gross Return	Benchmark Return	Current Month Excess Return	(Current Month Excess Return) - 0	((Current Month Excess Return) - 0)^2
1	4.6	0.5	4.1	4.1	16.81
2	4.6	0.5	4.1	4.1	16.81
3	4.6	0.5	4.1	4.1	16.81
4	4.6	0.5	4.1	4.1	16.81
5	4.6	0.5	4.1	4.1	16.81
6	4.6	0.5	4.1	4.1	16.81
7	4.6	0.5	4.1	4.1	16.81
8	4.6	0.5	4.1	4.1	16.81
9	4.5	0.4	4.1	4.1	16.81
10	4.6	0.5	4.1	4.1	16.81
11	4.6	0.5	4.1	4.1	16.81
12	4.5	0.4	4.1	4.1	16.81
Average	4.6	0.5	4.1		16.81
Standard Deviation			0.00		
Square Root					4.10
Total Return	71.2	6.0	65.3		
Annualized Deviation from Zero					14.20

PROBLEM #2—Excess Returns Do Not Reflect Wealth Relatives

On to the second problem that excess returns are usually expressed as arithmetic differences. The case below illustrates my point:

Table D: Geometric vs. Arithmetic Excess Returns*

Time Period	(1) Benchmark Return	(2) Manager A Return	(3) Manager B Return	(4) Manager A Arithmetic Excess Return	(5) Manager B Arithmetic Excess Return	(6) Manager A Geometric Excess Return	(7) Manager B Geometric Excess Return
Year 1	100%	80%	100%	-20%	0%	-10%	0%
Year 2	-50%	-50%	-70%	0%	-20%	0%	-40%
Cumulative	0%	-10%	-40%	-10%	-40%	-10%	-40%

* Column 4 = Column 2 - Column 1
Column 5 = Column 3 - Column 1
Column 6 = [(1 + Column 2) / (1 + Column 1)] - 1
Column 7 = [(1 + Column 3) / (1 + Column 1)] - 1

Think of geometric returns as representing wealth. Here is an illustration of what happens to an initial $1000 investment:

Table E: Manager A and B Cumulative Wealth Comparisons

Time Period	Benchmark Wealth	Manager A Wealth	Manager B Wealth
Beginning of Year 1	$1,000	$1,000	$1,000
Beginning of Year 2	$2,000	$1,800	$2,000
End of Year 2	$1,000	$900	$600

Most people in the financial community will look at arithmetic excess returns and describe both managers A and B as having underperformed by 20% one year and having matched the benchmark in the other year. However, as can be seen above, underperforming a down market hurts more than underperforming an up market from the standpoint of terminal wealth. This is because the excess return should be calculated on a geometric basis

((1+Rm)/(1+Rb))-1 where Rm = Return of the managed portfolio and Rb = Return of the benchmark. The traditional arithmetic excess return calculation just uses Rm – Rb.

Calculated using geometric excess returns it is clear that Manager A's performance in year 1 was far superior to Manager B's performance in year 2. Using geometric excess returns also makes Manager A's calculated risk much lower than Manager B's calculated risk. A manager that lost 10% to relative wealth in one year and performed in line with the market the other year is clearly less risky than the manager that lost 40% to relative wealth one year and performed in line with the market the other year.

RECOMMENDATION #2: Calculate geometric excess returns not arithmetic excess returns.

PROBLEM #3—Annualizing Tracking Error Can Give Misleading Results

On to the third problem that annualizing tracking error by multiplying by the square root of time can give misleading results, particularly when there is serial correlation in the return pattern. The case below illustrates my point:

Table F: Standard Deviation Calculation Example Repeated

Month Number	Gross Return	Benchmark Return	Current Month Excess Return	(Current Month Excess Return) - (Average of all Months' Excess Returns)	((Current Month Excess Return) - Average of All Months' Excess Returns))^2
1	3.2	0.5	2.7	(1.4)	1.96
2	2.4	0.5	1.9	(2.2)	4.84
3	6.2	0.5	5.7	1.6	2.56
4	1.2	0.5	0.7	(3.4)	11.56
5	5.3	0.5	4.8	0.7	0.49
6	3.5	0.5	3.0	(1.1)	1.21
7	8.3	0.5	7.8	3.7	13.69
8	4.1	0.5	3.6	(0.5)	0.25
9	3.5	0.4	3.1	(1.0)	1.00
10	4.2	0.5	3.7	(0.4)	0.16
11	8.8	0.5	8.3	4.2	17.64
12	4.2	0.4	3.8	(0.3)	0.09
Average	4.6	0.5	4.1		4.62
Standard Deviation			2.15		
Square Root					2.15
Total Return	70.6	6.0	64.7		
Annualized Standard Deviation			7.5		7.5
Information Ratio			8.7		

These are the same returns for the same manager shown earlier in the paper. Because the outperformance was consistent, annualizing monthly data implies that the risk to an investor is less than 8% per year. Appealing to logic and common sense, if a manager is taking enough risk to increase wealth by over 60% in one year, then it is also taking enough risk to decrease wealth by over 60% in one year. A better approach would be to use rolling 12-month returns.

RECOMMENDATION #3: Do not annualize monthly returns (or the returns of any period shorter than one year) to measure the risk of a strategy.

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A New Risk Measure Risk-K

Putting all three points together we have a new way to calculate risk called "Risk-K," a term coined by Chuck DuBois, one of the founders of Invesco Quantitative Strategies .

In the one period example illustrated earlier, the manager's 12-month return was 70.6% while the 12-month return of the benchmark was 6.0%. Thus, the wealth relative excess return (also referred to as the "geometric excess return") was (1.706 / 1.06)-1 = 60.9%.

In a multiple period example, take the average of the absolute value of all of the 12-month relative excess returns for the manager. In the case of this particular manager, using monthly returns from March 1994 to August 1997, the average of all of the rolling 12-month absolute wealth relative excess returns was 45.88%.

So, using the Risk-K measure would show this manager as being very risky. Using the traditional measure of annualized standard deviation that is used by the financial community, from March 1994 to August 1997, the same manager demonstrated risk of 11.97%.

Which measure is a better determinant of risk? Risk-K at 45.88% or annualized standard deviation at 11.97%?

Not to keep you in suspense any longer, the unnamed manager is Long-Term Capital Management (LTCM) and the 12-month period of returns is from March 1995 to February 1996. In retrospect, Risk-K did a better job of measuring risk, as LTCM went on to lose about 90% of the value of its investments during 1998. This event should not have occurred if annualized standard deviation had been an accurate measure of risk. Even assuming that LTCM had an expected excess return of zero, a 90% loss would be a -7.5 (-90% / 12% = -7.5) standard deviation event. A -7.5 standard deviation event should occur once every 31 trillion years, as is shown below.

Table G: The Chance of a (-X) Standard Deviation Event Occuring

Standard Deviation Event	% Chance of an X Standard Deviation Event	In how many occurrences does an X standard event happen?
(1.0)	15.865525393146%	6
(2.0)	2.275013194818%	44
(2.5)	0.620966532578%	161
(5.0)	0.000028665157%	3,488,556
(7.5)	0.000000000003%	31,339,202,408,202

On the other hand, using Risk-K, LTCM's disastrous period would have only been a 2 standard deviation event (-90% / 45.88%). Thus, the LTCM disaster could have been expected to happen once every 44 years

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Table H: Risk as Measured by Risk-K for Long-Term Capital Management**

Date	(1) LCTM Rolling 12-Month Return	(2) Benchmark Rolling 12-Month	(3) 12-Month Geometric Excess Return	(4) Absolute Return Calculation
02/95	35.38%	4.64%	29.38%	29.38%
03/95	41.27%	4.85%	34.73%	34.73%
04/95	42.66%	5.04%	35.81%	35.81%
05/95	41.86%	5.21%	34.83%	34.83%
06/95	49.39%	5.35%	41.81%	41.81%
07/95	40.95%	5.47%	33.64%	33.64%
08/95	40.55%	5.58%	33.12%	33.12%
09/95	52.82%	5.66%	44.64%	44.64%
10/95	57.51%	5.71%	49.00%	49.00%
11/95	51.37%	5.74%	43.16%	43.16%
12/95	59.00%	5.74%	50.38%	50.38%
01/96	62.44%	5.72%	53.64%	53.64%
02/96	70.62%	5.69%	61.43%	61.43%
03/96	67.65%	5.62%	58.73%	58.73%
04/96	81.89%	5.55%	72.33%	72.33%
05/96	74.18%	5.48%	65.13%	65.13%
06/96	83.48%	5.43%	74.03%	74.03%
07/96	77.55%	5.38%	68.49%	68.49%
08/96	71.03%	5.35%	62.35%	62.35%
09/96	69.14%	5.33%	60.59%	60.59%
10/96	61.99%	5.30%	53.83%	53.83%
11/96	58.55%	5.28%	50.59%	50.59%
12/96	57.48%	5.25%	49.62%	49.62%
01/97	53.57%	5.23%	45.94%	45.94%
02/97	48.27%	5.21%	40.93%	40.93%
03/97	49.29%	5.23%	41.87%	41.87%
04/97	36.79%	5.25%	29.97%	29.97%
05/97	40.16%	5.26%	33.15%	33.15%
06/97	29.11%	5.26%	22.66%	22.66%
07/97	28.99%	5.26%	22.54%	22.54%
08/97	30.41%	5.25%	23.90%	23.90%
Average of the absolute value of 12 month rolling geometric excess returns: Risk -K				45.88%

** Column 3 = [(1 + Column 1) / (1 + Column2)] - 1
Column 4 = Absolute Value of Column 3

Conclusion

While we wish that our industry would adopt a different, and more realistic, measure of risk, we doubt this will be the case, as a lot of time and effort have been devoted to incorporating standard deviation into the thinking of the average finance professional. On the other hand, it never hurts to try. In his book, "The Structure of Scientific Revolutions," Thomas S. Kuhn contended that changes in science do not happen smoothly. Rather, crisis triggers change (like the LTCM debacle), which makes practitioners realize that the current model (standard deviation as a measure of tracking error) needs to be replaced by a new paradigm. Even with the evidence of a crisis, financial professionals tenaciously hold on to the old paradigm, in which they have invested so much personal intellectual capital.

According to Kuhn, the first step in a paradigm shift occurs when an anomaly is introduced. In this paper, I have introduced the return pattern of LTCM as an anomaly. Hopefully, financial professionals will start to focus on new measures of risk: measures of risk that focus on the likelihood of having a downside event and the magnitude of that possible downside event. One possible measure is Risk-K.

Kuhn, Thomas S.,
The Structure of Scientific Revolutions, third edition, Chicago: University of Chicago Press, 1996.

Perold, Andre F.,
Long-Term Capital Management, L.P., Boston: Harvard Business School Publishing, November 5, 1999, exhibit 1 of the case on page 19.

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